OPTIMIZED COOPERATIVE SPECTRUM-SENSING IN CLUSTERED COGNITIVE RADIO NETWORKS

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ABSTRACT

In cognitive radio networks, the pertinent task of spectrum sensing at the Secondary Users (SUs) can be achieved when the SUs cooperate in order to make a final decision about the presence of a communicating Primary User (PU). In this paper, we study a two-hop relaying system in which SUs are grouped into D clusters. The SUs transmit a simple power function (parameterized by p) of their observationto a Fusion Centre (FC) using D orthogonal channels. The FC combines the receptions from cooperating nodes linearly. The goal of this work is to maximize the probability of detection over the parameters D (number of clusters), p (power function exponent), and w(linear combining coefficients) for a given false alarm probability. Overall, this work quantifies the advantages of optimal cooperation in primary detection in cognitive radio networks.

KEYWORDS

Wireless Networks, Relaying, Spectrum Sensing, Cognitive Radios, Cooperation, Fusion Centre.

1. INTRODUCTION

Wireless communication is progressing at an accelerated speed. Increasing variety of applications and features of wireless devices is leading to demands for higher and higher data rates. However, the bandwidth licensed to radio communication is limited. The infamous question is “How do we get better data rates under limited bandwidth requirements to meet the demand?”.Efficient spectrum utilization is the key to answer this question.

In 2002, Federal Communications Commission (FCC), the US government agency that regulates the use of frequency bands of the electromagnetic spectrum, indicated that the licensed frequency bands are unused 90% of the time[1]. In 2008, FCC ruled that unused portions of the RF spectrum will be made available for public use under certain conditions. In the light of this rule, spectrum efficiency can be improved if radio devices are equipped with technologies that take advantage of the licensed spectrum when it is unused. An emerging advanced solution for efficient spectrum utilization is the so-called cognitive radios.

A cognitive radio (CR) is a transceiver technology in which frequency spectrum is continuously sensed for unoccupied spaces. In a CR system, the primary user (PU) is the one who has licensed privilege to transmit in a particular frequency band and other users known as secondary users (SU) are the unlicensed users who desire to share the spectrum. The available unused frequency bands are called ‘spectrum holes’. SUs sense the spectrum for spectrum holes continuously. A CR is capable of not only sensing the spectrum, but also, monitoring, detecting and adapting its communication channel access. For example, a CRcan intelligently adjust its transmission parameters according to the availability in the frequency bands[2],[3]. CRtechnology has gained a

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lot of attention in the last decade. Currently, communication standards are adaptingthis technology [4].

Cooperative spectrum sensing is a scheme in which SUs cooperate with each other in a distributed or centralized manner, in order to make the decision about spectrum availability. This could be done via a Fusion Centre (FC). The SUs sense the channel for the presence of PUs and relay a function of their observations to the FC for a collective decision. The choice of this relaying is critical in order to optimize the overall performance at the FC.In the next subsection, we summarize the recent relevant work on cooperative spectrum sensing.

1.1. Cooperative Spectrum Sensing

In cooperative spectrum sensing, the SUs collaborate with each other in sensing the spectrum [5]. If optimized, cooperation reduces the power requirements at the SUsand improves the sensing performance even if it may introduce overhead for certain cases. In the case when SUs cooperate through a FC, every SU transmits its received signal to the FC that makes a decision about the presence of a PU based on the collective information from all the SUs.This is also called relay-assisted cooperative spectrum sensing [6].

The transmissions of the SUs to the fusion center could be on orthogonal channels [6], [7]. In this case, each SUforwards a function of their observation to the fusion center through and individualorthogonal channel similar to the well-known time-division multiple-access (TDMA), or frequency-division multiple-access (FDMA). On the other hand, transmissions of the SUs to the fusion center could be non-orthogonal, that is cooperating SUs transmit a function of their observation byusing the same channel. In the non-orthogonal channel model, it is assumed that SUs are synchronized so that the received signal in the fusion center is the coherent sumof transmitted signals by SUs[8], [9]. For orthogonal channels, the fusion center canuse various combining techniques of the received vector to obtain the final decision. It is shown in [10][11] that the probability of error for coherent orthogonal channel system will not improve with the increasing number of SUs. On the otherside, the performance of the non-orthogonal channel systemimproves with the increasing number of SUs due to the array gain [9], [10].

It iswell-known that in order to have an energy-efficient and reliable spectrum sensing, it is important for SUs to cooperate with each other when sensing for the PUs. However, one has to carefully weigh the trade-offs between the achievable *Cooperative Gain* and the incurred *Cooperative Overhead*[12]. In the case of orthogonal access between SUs and a FC, each radio is dedicated anorthogonal channel, and the requirement for bandwidth scales by the number of SUs. Then, the receptions from SUs at the FC are combined. In general, the linear combining techniquesare attractive, because, they are simple compared to non-linear techniques, and when the weighting coefficients are optimized, the improvement in the probability of detection at the fusion center is significant. Onthe other hand, in the case when non-orthogonal access is utilized from SUs to the FC, bandwidth requirements are negligible. Furthermore, the additive noise in the non-orthogonal channelis negligible, especially for large networks, compared to orthogonal channels since it is independent of the number of SUs. The gains due to optimized weighting coefficients in orthogonal channels and the independence of noises from the number of SUs in non-orthogonal channels posea trade-off. In order to optimizethis trade-off, one scheme proposed in [13]by the first author: group-orthogonal multiple access channel (MAC) approach for spectrum sensing. In group- orthogonal MAC, SUs utilize the available orthogonal channels in clusters, and each SU transmit to the FC the energy of its reception from the PUs. In [13], authors exploit the benefits of both orthogonal and non-orthogonal transmissions byfinding the optimal number of users that should be in an orthogonal group and the optimal linear weighting coefficients at the FC.

In this paper, we study optimal relaying function at SUs under different channel access schemes from the SUs to the FC. The considered cases are orthogonal, non-orthogonal and group-orthogonal multiple-access channels (MACs). In the group-orthogonal case, SUs are clustered into D groups that transmit on D orthogonal channels. In fact, orthogonal MAC and non-orthogonal MAC are special cases of group-orthogonal MAC when D=number of SUs, and D=1, respectively. The expressionsfor the probability of detection as a function of probability of false alarm under different group sizes and mappings arederived and analysed. This paper optimizes the performance over a set of relay functions and channel access schemes. This will help the SUs to make intelligent decisions when selectinghow and what to send to FC,for given a probability of false alarm in detecting the spectrum availability.

The rest of the paper is organized as follows. In Section 2, we give the problem formulation. In Section 3, we derive theoptimal number of groups and weighting coefficientunder certain assumptions. Simulationresults are given in Section 4. Finally, Section 5 concludesthe paper.

2. SYSTEM MODEL

We consider a cognitive radio network that is composed of aPU, multiple SUs and a FC-which could also be one of the SUs (see Fig. 1). Although the spectrum band under consideration is licensed to the PUs, they may or may not be transmitting during the considered time-slot. Hence, SUs need to decide whether thePUis idle (null hypothesis) or it is using the channel (alternative hypothesis) in order to utilize the band efficiently. In the considered set-up, the decisions are made cooperatively- that is each user makes decisions based on receptions from multiple SUs which also serve as relays. When acting as relays, each SU makes an observation, and transmits a signal based on solely its observation to the FC. We assume the FC combines the received signals linearly and makes final decision about the existence of the primary based on the combined signal. Linear combining at the FC is an attractive method primarily due to its simplicity. The two hypotheses: H_0 (no primary user exists) andH_1 (at least one primary user exist) form a binary hypothesis test given as below:

$$\begin{aligned} &H_0: \ x_i(k) = v_i(k) \qquad\qquad\quad i = 1,....M, \quad k = 1,....N \\ &H_1: \ x_i(k) = h_i s(k) + v_i(k) \qquad i = 1,....M, \quad k = 1,....N \end{aligned}$$

where$x_i(k)$ is the observed signal by the ith secondary user over N timeslots, $s(k)$ is the transmitted signal by the PU in the kth timeslot and $v_i(k)$is the additive noise at the ith user in the kth timeslot. The noise$v_i(k)$ is assumed be white Gaussian noise with zero mean and variance $\sigma^2$and also $v_i(k)$sare assumed to be independent and identically distributed (i.i.d.) over time index k and user index i. The channel gains from the PU to the SUsare assumed to stay constant over the observation interval (slow fading scenario).

In the network, each SU observes the channel for N timeslots and then forwards a power function of the observed signal to the FC: $u_i = f(x_i) = \beta_i \, |x_i|^p$ where $\beta_i = P_i / \sqrt{E\{|x_i|^2\}}$ *is* the scaling factor so that average transmission power is boundedbythe power constraint P_i. A common relay operation is to send the energy of the observedsignal [13], which is equivalent to the case when p

is chosen to be 2, and β_i =1 in our scenario. Our goal is to optimize over the power function exponent p so that the performance is improved.

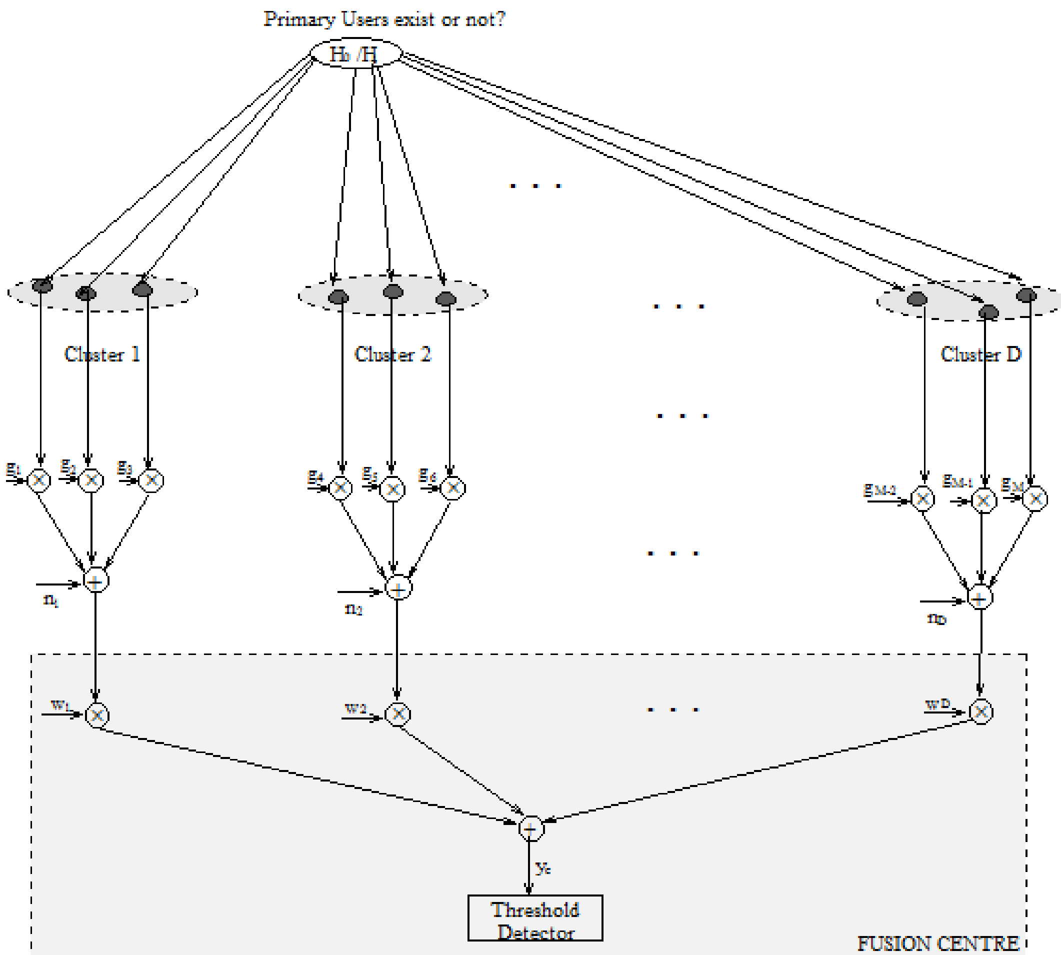

Figure 1. Group-orthogonal MAC with M = 3D secondary users and D clusters

In the network, SUsare grouped into *D*clusters (see Fig. 1). Each cluster is dedicated to on an orthogonal channel, and users in the same cluster transmit on the same orthogonal channel. We call this set-up *group-orthogonal MAC (Multiple Access Channel)*. The clusters are assumed to be pre-determined. For example, one could form clusters based on geographical proximity or signal quality. However, the question of how the clusters are formed is out of scope of this paper. Let S_j denote a set of users in the jth group where $j=1\ldots D$. For simplicity in the analysis, we also assume that clusters have equal number of nodes. Then, the combined signal in the jth orthogonal channel can be written as:

$$y_j = \sum_{m \in S_j} g_m u_m + n_j \qquad \mathrm{j} = 1,\ldots.D$$

where g_m is the channel gain from SU to the FC and n_j is the noise added at each channel and is assumed to be i.i.d. white Gaussian noise with zero mean and variance σ^2. When information from each group reaches the FC, it is linearly combined after being weighted. The weighting vector is defined as $\mathbf{w} = [w_1, w_2, \ldots, w_D]$.

After combining, the signal observed at the FC is denoted by:

$$y_c = \sum_{j=1}^{D} w_j y_j = \sum_{j=1}^{D} w_j \left(\sum_{m \in S_j} g_m u_m \right) + \sum_{j=1}^{D} n_j w_j$$

At the FC, the global test statistic $y_c$is compared with γ_c to make a decision about PUs, that is

$$\text{if } y_c \geq \gamma_c, \text{ decide that } H_1 \text{ has occurred,}$$
$$\text{if } y_c < \gamma_c, \text{ decide that } H_0 \text{ has occurred.}$$

3. OPTIMIZED COOPERATIVE SPECTRUM-SENSING

In this section, first we describe the performance metrics that are used, and then we describe the optimization problem. Solution for the optimization problem is also provided.

3.1. Performance Metrics

We use the two important metrics: probability of detection $P_d = P(H_1|\ H_1)$ and probability of false alarm $P_f = P(H_1|\ H_0)$. Our goal is to maximize the probability of detection, P_d, for a given probability of false alarm,P_f. The optimization is over the set of parameter: as the number of orthogonal channels D, therelaying function $\beta_i\ |x_i|^p$, and weighting coefficients w_js.

In the following, we will use central limit theory [14] to derive analytical expressions for $P_d$and P_f. We can argue that for large N (the observation time interval), x_i can be assumed to be asymptotically normally distributed as well as y_js and y_c. For a normally distributed random variabley_c, the probability of detection and false alarm can be expressed as follows for a given detection threshold $\gamma_c$at the FC:

$$P_d = P[H_1 \mid H_1] = P[y_c \geq \gamma_c \mid H_1] = Q\left[\frac{\gamma_c - E\left[y_c \middle| H_1 \right]}{\sqrt{Var\left[y_c \middle| H_1 \right]}} \right] \tag{1}$$

$$P_f = P[H_1 \mid H_0] = P[y_c \geq \gamma_c \mid H_0] = Q\left[\frac{\gamma_c - E\left[y_c \middle| H_0 \right]}{\sqrt{Var\left[y_c \middle| H_0 \right]}} \right] \tag{2}$$

where $Q(x) = 1/\sqrt{2\pi}\int_x^{\infty} e^{-u^2/2} du$ denotes the Q-function.

In order to find detection and false alarm probabilities based on the above mentioned formulas it is required to find the conditional mean and variances under both hypotheses for a given parameter set. Note that the P_d is actually a function of not only P_f,but also a function network parameterssuch as the number of cluster (D), the relay function exponent (p), the FC combining coefficients ($\boldsymbol{w}$), channel coefficients between primary and cognitive radios (h_is), the channel coefficients between the cognitive radios and the FC (g_is), transmission power of the radios (P_is), and the noise powers $\sigma^2$and δ^2. Our goal is to maximize the probability of detection over the parameters: (i) the number of clusters (D); *(ii)*the relaying function exponent (p); and (iii) weighting coefficients at the FC, $\boldsymbol{w}$, when the rest of the parameters are given.This implies that the channel state information (CSI) is known for both links between PUs and SUs, and SUs and the FC. Note that the channel fading coefficients h_is and g_is are assumed to be slowly varying; hence the CSI assumption is sensible. The following lemma provides an explicit analytical expression for P_d as a function of P_f.

Lemma 1: For given probability of false alarm P_f, number of clusters D, relaying function exponent p, and weighting coefficients w, if the channel gains $|h_i|^2$s are all equal ($|h_i|^2 = \alpha, \forall i$), then the probability of detection P_d is given as follows for large N:

$$P_d(D,p,\boldsymbol{w}) = Q\left[\frac{E\left[y_c|H_0\right] - E\left[y_c|H_1\right] + Q^{-1}\left(P_f\right)\sqrt{Var\left[y_c|H_0\right]}}{\sqrt{Var\left[y_c|H_1\right]}}\right] \tag{3}$$

where

$$E[y_c\,|\,H_0] = A_p \boldsymbol{g}_D^H \boldsymbol{w},\quad E[y_c\,|\,H_1] = B_p \boldsymbol{g}_D^H \boldsymbol{w},$$
$$\mathrm{Var}[y_c\,|\,H_0] = C_p \boldsymbol{w}^H \boldsymbol{G}_D\, \boldsymbol{w} + \delta^2 \boldsymbol{w}^H \boldsymbol{w},$$
$$\mathrm{Var}[y_c\,|\,H_1] = D_p \boldsymbol{w}^H \boldsymbol{G}_D\, \boldsymbol{w} + \delta^2 \boldsymbol{w}^H \boldsymbol{w},$$

and

$$\boldsymbol{g}_D = [\sum_{i\in S_1} g_i \sqrt{P_i}, \ldots, \sum_{i\in S_D} g_i\sqrt{P_i}],\qquad \mathrm{G}_D = \mathrm{diag}(\sum_{i\in S_1} g_i^2 P_i, \ldots, \sum_{i\in S_D} g_i^2 P_i).$$

The coefficients A_p, B_p, C_p, and D_p depend on the relay function exponent p and are given as follows:

$$A_p = \Gamma\left(\frac{p+1}{2}\right) \Big/ \sqrt{\frac{\sqrt{\pi}}{N}\Gamma\left(\frac{2p+1}{2}\right) + \frac{N-1}{N}\,\Gamma^2\left(\frac{p+1}{2}\right)}$$

$$B_p = \left(\sum_{k=0}^{N-1}\Phi_{p,k}\right) \Big/ \sqrt{\pi^{1/2}\,\frac{\Gamma\left(\frac{2p+1}{2}\right)}{\Gamma^2\left(\frac{p+1}{2}\right)}\left(\sum_{k=0}^{N-1}\Phi_{2p,k}\right) + 2\sum_{k_1<k_2}\Phi_{p,k_1}\,\Phi_{p,k_2}}$$

$$C_p = \left(\Gamma\left(\frac{2p+1}{2}\right) - \frac{1}{\sqrt{\pi}}\Gamma^2\left(\frac{p+1}{2}\right)\right) \Big/ \left(\Gamma\left(\frac{2p+1}{2}\right) + \frac{N-1}{\sqrt{\pi}}\,\Gamma^2\left(\frac{p+1}{2}\right)\right)$$

$$D_p = \left(\Gamma\left(\frac{2p+1}{2}\right)\sum_{k=0}^{N-1}\Phi_{2p,k} - \frac{1}{\sqrt{\pi}}\Gamma^2\left(\frac{p+1}{2}\right)\sum_{k=0}^{N-1}\Phi_{p,k}^2\right) \Big/ \left(\Gamma\left(\frac{2p+1}{2}\right)\sum_{k}\Phi_{2p,k} + \frac{2}{\sqrt{\pi}}\Gamma^2\left(\frac{p+1}{2}\right)\sum_{k_1<k_2}\Phi_{p,k_1}\Phi_{p,k_2}\right)$$

where $\Phi_{p,k} = {}_1F_1\left(\frac{-p}{2}, \frac{1}{2}, \frac{-\alpha\, s^2(k)}{2\sigma^2}\right)$, $\Gamma(x)$ denotes the gamma function, and ${}_1F_1$ denotes the confluent hyper-geometric function[15].

Proof: Using Eqn. (2), for any givenP_f, threshold $\gamma_c$can be written as:

$$\gamma_c = E\left[y_c|H_0\right] + Q^{-1}\left(P_f\right)\sqrt{Var\left[y_c|H_0\right]} \tag{4}$$

By substituting Eqn. (4) in Eqn. (1), we obtain Eqn. (3). The derivations for $E[y_c|H_0]$, $E[y_c|H_1]$, $Var[y_c|H_0]$, *and* $Var[y_c|H_1]$ are given in the Appendix 3.1. ♦

It is important to note that Lemma 1 provides a formulation in which the dependences on the optimization parameters (D, p, $\boldsymbol{w}$) are partially decoupled. The variables $\boldsymbol{w}$, $\boldsymbol{g}_D$ and $\boldsymbol{G}_D$ depend on the cluster size D, the constants A_p, B_p, C_p, and D_p depend on the relay function exponent p, and the weighting coefficient $\boldsymbol{w}$ shows up explicitly in the expression. This will help solve the optimization problem.

3.2. Optimized Cooperative Transmission and Reception

We formulate the optimization problem as follows. Given the channel gains ($|h_i|^2$s, and g_is), the transmission powers of the SUs (P_is), and the noise powers $\sigma^2$and δ^2, the goal is to maximize the P_d for a given limit on false alarm probabilityP_f:

$$\max_{D,p,\boldsymbol{w}} \; P_d(D,p,\boldsymbol{w})$$

We make the following assumptions in order to solve this problem.

1. Uniform channel gains: $|h_i|^2 = \alpha$, and $g_i = \beta > 0$.
2. Uniform transmission powers: $P_i = P$for all i.
3. The orthogonal groups have equal number of SUs (assuming M/D is an integer).
4. The weighting coefficients,w_is,are nonnegative.

Under these assumptions the following theorem provides the optimal D,and $\boldsymbol{w}$for a given p value.

Theorem 1: In the group-orthogonalMAC system, for a given relay power function with exponent p, if channelgains are equal ($|h_i|^2 = \alpha, g_i=\beta$), then the optimalweighting coefficients that maximizes P_d(Eqn. (3)) are uniform for a given D, that is

$$w_i = 1/D,\ i = 1 \ldots D.$$

And the optimal$D$ for given $\boldsymbol{w}$ and p is given as

$$D \cong \begin{cases} 1 & P_f > Q\left(\dfrac{(A_p - B_p)\sqrt{C_p\beta^2 PM + \delta^2}}{(D_p - C_p)\beta\sqrt{P}}\right) \\ M & P_f < Q\left(\dfrac{(A_p - B_p)\sqrt{C_p\beta^2 PM + \delta^2 M}}{(D_p - C_p)\beta\sqrt{P}}\right) \\ \left.\underline{\dfrac{Q^{-2}(f)(D_p - C_p)^2\beta^2 P}{(A_p - B_p)^2\ \delta^2} - \dfrac{C_p\beta^2 PM}{\delta^2}}\right|_M & \text{otherwise} \end{cases}$$

where $\left.\underline{X}\right|_M$ denotes the divisor of M that is closed to X.

Proof: See Appendix 3.2. ♦

The theorem states optimal linear combining coefficients at the FC should be uniform for D orthogonal channels and should sum to 1. This is very intuitive due to the assumptions on the equal channel gains and noise powers, and also identical relay functions at the relays. On the other hand, optimal Dhas three different regions: (i) when the false alarm probability is high ($P_f > P_L$), the optimal $D$is equal to 1 which implies that non-orthogonal transmission is optimal; (ii) when the false alarm probability is low ($P_f < P_H$), the optimal $D$is equal to M which implies that each SU should transmit on an orthogonal channel and no clustering of SUs; and (iii) when the false alarm probability is between P_L and P_H, optimal scheme is group-orthogonal transmission. Note that for some special scenarios, the third region may merge with one of the other regions, that is the rounding operation $\rfloor_M$ in the above equation may lead to D=1 or D=M.

Optimization over the relaying function exponent p can be done by replacing the optimal values for D and **w** obtained in Theorem 1 in Eqn. 3, and by using an optimization toolbox for nonlinear integer programming.

4. SIMULATIONS

In this section, we provide probability of detection versus probability of false alarmcurves for different values of p and D. For all the simulations, the number of users M=4, observation time N=1, the channel gains $|h_i|^2 = \alpha = 100$, $g_i = \beta = 1$, relay transmission powers P =1, and noise powers $\sigma^2 = 1$, and $\delta^2 = 5$. Below,P_d denotes the probability of detection and P_f denotes the probability of false alarm. We assume the primary signal $|s(k)|^2 = 1/N$, for all k, for simplicity.

In Fig.2, we display the P_d as a function of P_f when the relay function has exponent $p=1$ and $p=3$. The curves for various channel access scenarios between relays and FCare shown: orthogonal access ($D=M=4$), non-orthogonal access ($D=1$), and group-orthogonal access ($D=2$). It is observed that for lower probability of false alarms, orthogonal MACgives the best probability of detection, and for higher probability of false alarms, non-orthogonal MACgives the best probability of detection. Using Theorem 1, we can obtain the boundaries of these two different regions:$P_f > 0.2797$ and $P_f < 0.2180$ for $p=1$ and $P_f > 0.1180$ and $P_f < 00885$ for $p=3$, which are consistent with the simulations.In Fig. 3, we display the zoomed curves corresponding to the region $0.2180 < P_f < 0.2797$for $p=1$. According to Theorem 3, in this region optimal D could be 1, 2, and 4 which is what we observe in Fig. 3. Overall, the relay function with exponent p=3 outperforms the relay function with p=1. Furthermore, we observe that therange of P_f where group-orthogonal MAC is optimal is getting smaller with the increase in p.

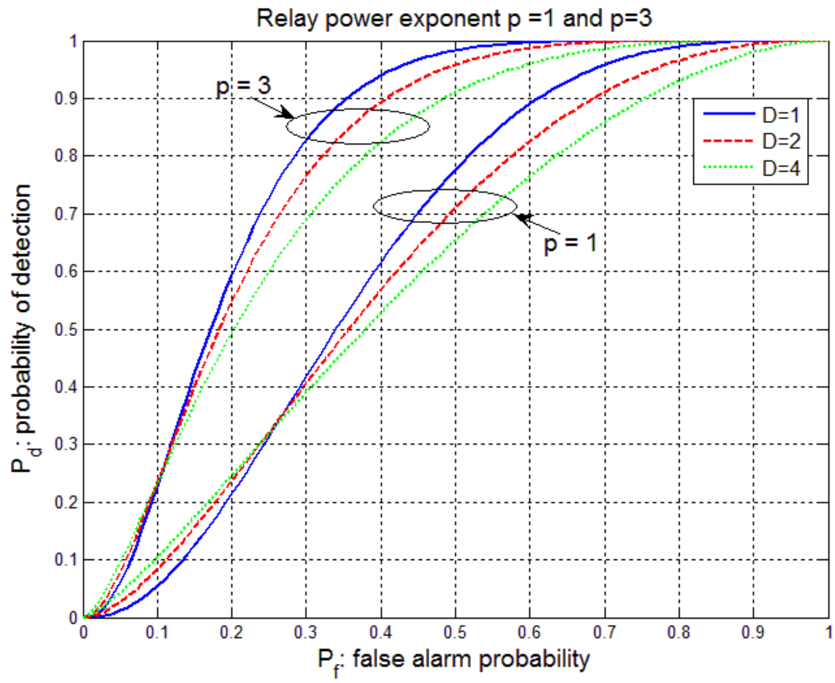

Figure 2 Probability of detection (P_d) vs. Probability of false alarm (P_f) with p =1 and p=3

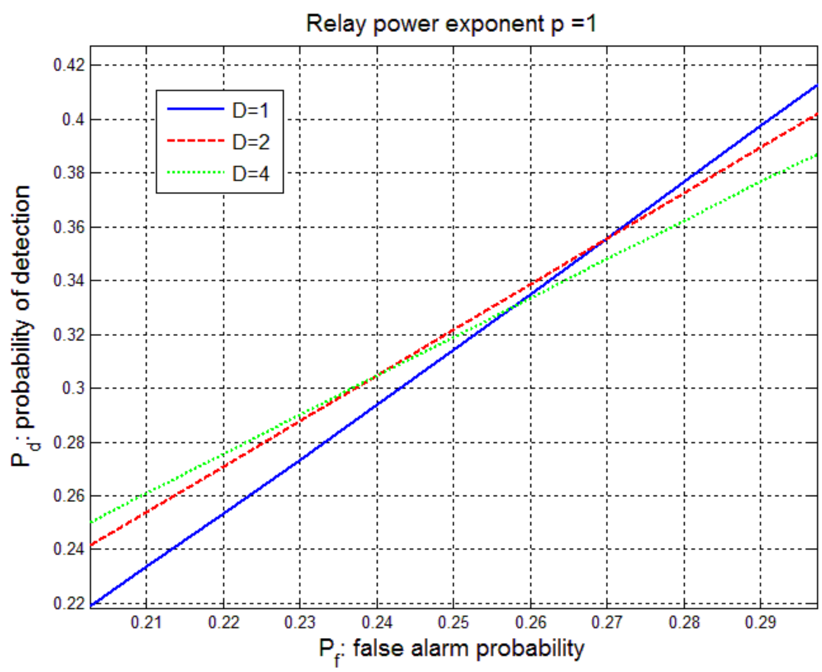

Figure 3 P_d vs. P_f for different ranges of P_f: ($0.21<P_f<0.29$)

Next, we analyse different relay functions for a given channel access scheme in detail. In Fig. 4 and Fig.5, we plot the P_d vs. P_f curves for the non-orthogonal MAC (D=1), and orthogonal MAC(D=M=4), respectively.It can be concluded that for a given D, there does not exist a single relay function that performs optimally for all P_f values. In the limit where relay function exponent p is large, the curves reduces to $P_d = P_f$ line for any Dvalue. Similar behaviour is observed for other D value.

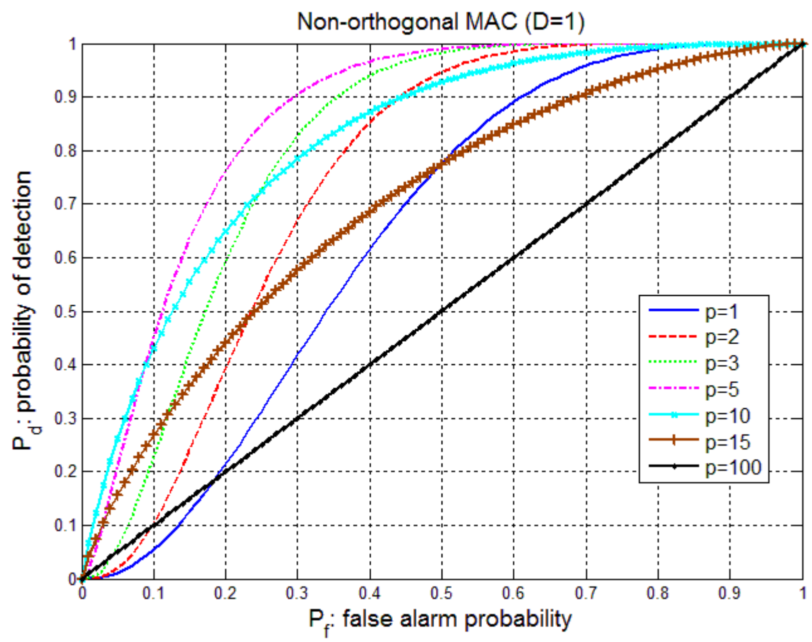

Figure 4P_d vs. P_f for non-orthogonal channel (D = 1)

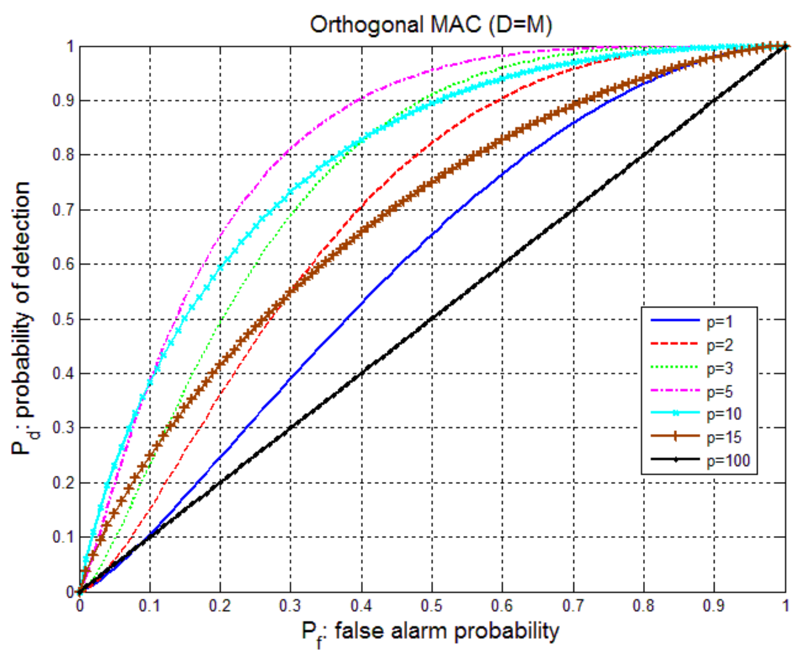

Figure 5Pd vs. Pf for orthogonal channel (D = M)

In Table 1, we display the optimal relay function exponent and optimal cluster size D for various P_f values. It is important to note that optimal $D$is always equal to 1, which implies that when optimal relay function (or equivalently p) *is* selected for a given false alarm probability, then the non-orthogonal scheme becomes optimal globally. In addition, as $P_f$increases, the optimal relay function exponent p decreases.

Table 1 Optimal D and p

Range of P_f	***Optimal p***	***Optimal D***
$0 < P_f < 0.001$	>11	1
$0.001 < P_f < 0.003$	11	1
$0.004 < P_f < 0.009$	10	1
$0.010 < P_f < 0.026$	9	1
$0.027 < P_f < 0.059$	8	1
$0.060 < P_f < 0.126$	7	1
$0.127 < P_f < 0.250$	6	1
$0.251 < P_f < 0.487$	5	1
$0.488 < P_f < 0.897$	4	1
$0.898 < P_f < 0.998$	3	1
$0.999 < P_f < 1.000$	2	1
$P_f = 1$	1	1

Overall, the optimal relay function for any of the channel access schemes is always the power function with high exponents for lower probability of false alarms.However, if the system is robust enough to handle higher probability of false alarms, small power exponents such as p =1,2 or 3is the optimal choice of for the relay functions.It can also be concluded that globally non-orthogonal scheme is optimal under the given assumptions.

5. CONCLUSIONS

In this paper, we studied a cognitive radio network in which SUs cooperate in order to make a decision about the primary existence. The proposed scheme is distributed in the sense that the cooperating SUs transmit a power function (parameterized with exponent p) of their local observation, hence does not require any overhead due to cooperation. The SUs transmit to aFC (which could also be one of the SUs) over D orthogonal channels, and FCcombines these receptions linearly using weighting coefficients $\boldsymbol{w}$. We provided analytical solutions and simulations for maximizing the probability of detection at the fusion centre for a given false alarm probability over the parameters D, p, and $\boldsymbol{w}$. It is interesting that non-orthogonal channel access becomes optimal globally when the best relaying function is utilized even though the orthogonal or group-orthogonal access schemes require more bandwidth. This behaviour is not observed in cooperation strategies whererelays simply send their energy to the fusion centre [13]. In summary, this work shows the importance of optimization in cooperative cognitive radio networks in order to extract the gains of cooperation for spectrum sensing with negligible overhead.

6. APPENDICES

6.1. Proof of Lemma 1

Derivation of $E\left[y_c \,|H_0\right]$**:** For the first hypothesis H_0 we derive the expected value as:

$$E\left[\sum_{k=0}^{N-1}\left|x_i\,(\mathrm{k})\right|^p |H_0\right] = E\left[\sum_{k=0}^{N-1}\left|v_i\,(k)\right|^p\right] = \sum_{k=0}^{N-1} E\left[\left|v_i\left(k\right)\right|^p\right]$$

We know that $v_i(k)$ is a zero-mean Gaussian $N\left(0,\sigma_i^2\right)$ and for such a random variable the expected value of the absolute function is given by [16]:

$$E\left[|\boldsymbol{X}|^p\right]=\frac{1}{\sqrt{\pi}}\ \sigma^p.2^{\frac{p}{2}}.\Gamma\left(\frac{p+1}{2}\right)$$

Hence,

$$\sum_{k=0}^{N-1}E\left[\left|v_i(k)\right|^p\right]=\sum_{k=0}^{N-1}\frac{\sigma^p 2^{\frac{p}{2}}\Gamma\left(\frac{p+1}{2}\right)}{\sqrt{\pi}}=\frac{N}{\sqrt{\pi}}\ \sigma^p 2^{\frac{p}{2}}\Gamma\left(\frac{p+1}{2}\right) \tag{5}$$

Derivation of Normalization factor for Hypothesis H_0: All the above derived expressions are yet not normalized there we need to find expressions for the normalization factors for every hypothesis. The normalization factor hypothesis H_0 can be derived as follows:

$$\sqrt{E\left[\left(\sum_{k=0}^{N-1}\left|x_i\,(\mathrm{k})\right|^p\right)^2\Big|H_0\right]}=\sqrt{\mathrm{E}\left[\left(\sum_{k=0}^{N-1}\left|v_i\,(\mathrm{k})\right|^p\right)^2\right]}$$

It can be easily derived that the expected value of square of sums is given by:

$$E\left[\left(\sum_{k=0}^{N-1}\left|\boldsymbol{X}_k\right|^p\right)^2\right]\ =\ \sum_{k=0}^{N-1}E\left[\left|\boldsymbol{X}_k\right|^{2p}\right]+\sum_{k_1<k_2}E\left[\left|\boldsymbol{X}_{k_1}\right|^p\right]E\left[\left|\boldsymbol{X}_{k_2}\right|^p\right] \tag{6}$$

Since $v_i(k)$ is independent and identically distributed for each k, substituting the values of the expectations in the above equation gives:

$$E\left[\left(\sum_{k=0}^{N-1}\left|x_i\,(\mathrm{k})\right|^p\right)^2\Big|H_0\right]=\frac{N}{\sqrt{\pi}}\sigma^{2p}2^p\Gamma\left(\frac{2p+1}{2}\right)\ +\ \frac{N(N-1)}{\pi}\sigma^{2p}2^p\Gamma^2\left(\frac{p+1}{2}\right) \tag{7}$$

Since y_c is a linear combination of x_i after being scaled by channel gain and weighting factor, so the expected value of y_c can be written as:

$$E\left[y_c\,|H_0\right]=\frac{\frac{N}{\sqrt{\pi}}\ \sigma^p 2^{\frac{p}{2}}\Gamma\left(\frac{p+1}{2}\right)}{\sqrt{\frac{N}{\sqrt{\pi}}\sigma^{2p}2^p\Gamma\left(\frac{2p+1}{2}\right)\ +\ \frac{N(N-1)}{\pi}\sigma^{2p}2^p\Gamma^2\left(\frac{p+1}{2}\right)}}\ \mathbf{g}^H\mathbf{w} \tag{8}$$

Derivation of $E\left[y_c\,|H_1\right]$: For the second Hypothesis, H_1 we derive the expected value as

$$E\left[\sum_{k=0}^{N-1}\left|x_i\,(\mathrm{k})\right|^p\,|H_1\right]=\sum_{k=0}^{N-1}E\left[\left|h_i s(k)+v_i(k)\right|^p\right]$$

We see that $x_i(k)$ is a non-zero mean Gaussian random variable $N(h_i s(k), \sigma^2_i)$ and for a Gaussian random variable with mean μ_x and variance σ^2_x

$$E\left[|\boldsymbol{X}|^p\right]=\frac{\sigma_x{}^p.2^{\frac{p}{2}}.\Gamma\left(\frac{p+1}{2}\right)}{\sqrt{\pi}}\cdot{}_1F_1\left(\frac{-p}{2},\frac{1}{2},\frac{-1}{2}\left(\frac{\mu_x}{\sigma_x}\right)^2\right)$$

where ${}_1F_1$ is the confluent hyper geometric function.Hence,

$$\sum_{k=0}^{N-1} E\left[\left|h_i s(k)+v_i(k)\right|^p\right] = \sum_{k=0}^{N-1}\frac{\sigma^p .2^{\frac{p}{2}}.\Gamma\left(\frac{p+1}{2}\right)}{\sqrt{\pi}}\,{}_1F_1\left(\frac{-p}{2},\frac{1}{2},\frac{-1}{2}\frac{\left|h_i\right|^2.\left|s(k)\right|^2}{\sigma^2}\right)$$

Using the assumptions we have made about the channel gain h_i, we can simplify the expression as following:

$$\sum_{k=0}^{N-1} E\left[\left|h_i s(k)+v_i(k)\right|^p\right] = \frac{1}{\sqrt{\pi}}\ \sigma^p 2^{\frac{p}{2}}\Gamma\left(\frac{p+1}{2}\right)\sum_{k=0}^{N-1}{}_1F_1\left(\frac{-p}{2},\frac{1}{2},\frac{-\alpha\ s^2(k)}{2\sigma^2}\right)$$

Derivation of Normalization factor for Hypothesis H_1: Using(6), the normalization factor hypothesis H_1 can be derived as follows:

$$E\left[\left(\sum_{k=0}^{N-1}\left|x_i\,(\mathrm{k})\right|^p \left|H_1\right.\right)^2\right] = \sum_k \frac{\sigma^{2p}2^{\frac{2p}{2}}\Gamma\left(\frac{2p+1}{2}\right)}{\sqrt{\pi}}\,{}_1F_1\left(\frac{-2p}{2},\frac{1}{2},\frac{-\alpha\ s(k)}{2\sigma^2}\right)+$$
$$\sum_{k_1<k_2}\left(\sigma^p 2^{\frac{p}{2}}\Gamma\left(\frac{p+1}{2}\right)\frac{1}{\sqrt{\pi}}\right)^2 {}_1F_1\left(\frac{-p}{2},\frac{1}{2},\frac{-\alpha s(k_1)}{2\sigma^2}\right){}_1F_1\left(\frac{-p}{2},\frac{1}{2},\frac{-\alpha s(k_2)}{2\sigma^2}\right) \tag{9}$$

Therefore, we can write the $E\left[y_c\left|H_1\right.\right]$ as:

$$E\left[y_c\left|H_1\right.\right]=\frac{\frac{1}{\sqrt{\pi}}\displaystyle\sum_{k=0}^{N-1}{}_1F_1\left(\frac{-p}{2},\frac{1}{2},\frac{-\alpha\ s^2(k)}{2\sigma^2}\right)}{\sqrt{\frac{1}{\sqrt{\pi}}\frac{\Gamma\left(\frac{2p+1}{2}\right)}{\Gamma^2\left(\frac{p+1}{2}\right)}\displaystyle\sum_{k=0}^{N-1}{}_1F_1\left(\frac{-2p}{2},\frac{1}{2},\frac{-\alpha\ s(k)}{2\sigma^2}\right)+\frac{2}{\pi}\sum_{k_1<k_2}\Phi_{p,k_1}\,{}_1\Phi_{p,k_2}}}\mathbf{g}^H\mathbf{w} \tag{10}$$

where

$$\Phi_{p,k} = {}_1F_1\left(\frac{-p}{2},\frac{1}{2},\frac{-1}{2},\frac{-\alpha\ s^2(k)}{2\sigma^2}\right). \tag{11}$$

Derivation of $Var\left[y_c\left|H_0\right.\right]$**:** For the first Hypothesis, H_0 we derive the variance value as

$$Var\left[\sum_{k=0}^{N-1}\left|x_i\,(\mathrm{k})\right|^p\left|H_0\right.\right] = \sum_{k=0}^{N-1}Var\left[\left|v_i(k)\right|^p\right] = \sum_{k=0}^{N-1}\left(E\left[\left|v_i(k)\right|^{2p}\right]-\left(E\left[\left|v_i(k)\right|^p\right]\right)^2\right)$$

$$=\frac{N}{\sqrt{\pi}}\sigma^{2p}2^p\Gamma\left(\frac{2p+1}{2}\right)-N\left(\sigma^p 2^{\frac{p}{2}}\Gamma\left(\frac{p+1}{2}\right)\frac{1}{\sqrt{\pi}}\right)^2 \tag{12}$$

Using this expression and power normalization factor(7), we can find the $Var\left[y_c\left|H_0\right.\right]$ as:

$$Var\left[y_c\left|H_0\right.\right]=\left(\frac{\frac{N}{\sqrt{\pi}}\sigma^{2p}2^p\Gamma\left(\frac{2p+1}{2}\right)-N\left(\sigma^p 2^{\frac{p}{2}}\Gamma\left(\frac{p+1}{2}\right)\frac{1}{\sqrt{\pi}}\right)^2}{\frac{N}{\sqrt{\pi}}\sigma^{2p}2^p\Gamma\left(\frac{2p+1}{2}\right)+\frac{N(N-1)}{\pi}\sigma^{2p}2^p\Gamma^2\left(\frac{p+1}{2}\right)}\right)\mathbf{w}^H\mathbf{G}\mathbf{w}+\partial^2\mathbf{w}^H\mathbf{w}$$

(13)

Derivation of $Var\left[y_c\left|H_1\right.\right]$**:** For the second Hypothesis, H_1 we derive the variance value as

$$Var\left[\sum_{k=0}^{N-1}\left|x_i\,(\mathrm{k})\right|^p\left|H_1\right.\right]=\sum_{k=0}^{N-1}\left(E\left[\left|h_is(k)+v_i(k)\right|^{2p}\right]-\left(E\left[\left|h_is(k)+v_i(k)\right|^p\right]\right)^2\right)$$

$$=\sum_{k=0}^{N-1}\left(\frac{1}{\sqrt{\pi}}\sigma^{2p}2^p\Gamma\left(\frac{2p+1}{2}\right){}_1F_1\left(-p,\frac{1}{2},\frac{-\alpha s(k)}{2\sigma^2}\right)-\left(\frac{1}{\sqrt{\pi}}\sigma^p 2^{\frac{p}{2}}\Gamma\left(\frac{p+1}{2}\right){}_1F_1\left(-\frac{p}{2},\frac{1}{2},\frac{-\alpha s(k)}{2\sigma^2}\right)\right)^2\right)$$

(14)

Using the power normalization factor in Eqns.(9), (14), and (11), we obtain

$$Var\left[y_c\left|H_1\right.\right]=\left(\frac{\frac{1}{\sqrt{\pi}}\sigma^{2p}2^p\left(\Gamma\left(\frac{2p+1}{2}\right)\sum_{k=0}^{N-1}\Phi_{2p,k}-\frac{1}{\sqrt{\pi}}\Gamma^2\left(\frac{p+1}{2}\right)\sum_{k=0}^{N-1}\Phi_{p,k}^2\right)}{\frac{1}{\sqrt{\pi}}\sigma^{2p}2^p\left(\sum_k\Gamma\left(\frac{2p+1}{2}\right)\Phi_{2p,k}+\sum_{k_1<k_2}\Gamma^2\left(\frac{p+1}{2}\right)\frac{1}{\sqrt{\pi}}\Phi_{p,k_1}\Phi_{p,k_2}\right)}\right)\mathbf{w}^H\mathbf{G}\mathbf{w}+\partial^2\mathbf{w}^H\mathbf{w}$$

(15)

6.2. Proof of Theorem 1

Under the given assumptions P_{d} simplifies as

$$P_d=Q\left(\frac{(A_p-B_p)\left(\frac{\beta\sqrt{P}M}{D}\right)\left(\sum_{i=1}^{D}w_i\right)+Q^{-1}(P_f)\sqrt{\left(C_p\left(\frac{\beta^2PM}{D}\right)+\delta^2\right)\left(\sum_{i=1}^{D}w_i^2\right)}}{\sqrt{\left(D_p\left(\frac{\beta^2PM}{D}\right)+\delta^2\right)\left(\sum_{i=1}^{D}w_i^2\right)}}\right)$$

This formulation of P_d shows that P_d is a function of Σw_{i} and Σw_{i}^2. However, note that P_d is independent of Σw_{i}. This can be shown easily by replacing $\gamma\mathbf{w}$ instead $\mathbf{w}$. Then, P_d becomes independent of γ, hence we can claim that optimal **w** is such that Σw_{i} =1.Furthermore, using the above equation, we can see that the P_d is maximized when is Σw_{i}^2 minimized assuming Σw_{i} =1. This is achieved when **w** = (1/D)[1 … 1].

Assuming **w**= (1/D)[1 … 1], we can take the derivative of P_d wrt.*D* and find the optimal *D* when *D* ϵ {1,2, … ,M}. Note that *D*is an integer, and one has to pay attention to the boundary of the set {1,2, … ,M} while finding the *D* that maximized P_d. This operation will give us the optimal solution since *D* should be an integer.

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